

Virtual Photon Structure Functions *

T. Uematsu^a

^aDept. of Fundamental Sciences, FIHS, Kyoto University, Kyoto 606-8501, Japan

We discuss the perturbatively calculable virtual photon structure functions. First we present the framework for analyzing the structure functions of the virtual photon and derive a first moment of g_1^γ of the virtual photon. We then investigate the three positivity constraints satisfied by the eight structure functions of the virtual photon.

1. INTRODUCTION

I would like to talk about the virtual photon structures, especially about the general virtual photon structure functions and their positivity constraints. This work has been done in collaboration with Ken Sasaki and Jacques Soffer [1].

The structure of the virtual photon can be studied from the double-tagged two-photon processes in e^+e^- collisions (Fig.1) or the 2-jet events in resolved photon processes of $e^{-(+)}p$ collisions.

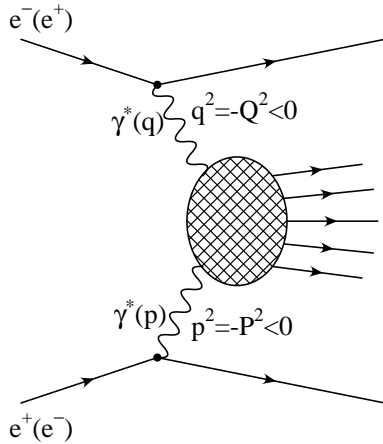


Figure 1. Two-photon process in e^+e^- collision

The experimental data provide the effective photon structure functions, $F_{\text{eff}}^\gamma \simeq F_{TT}^\gamma + F_{TL}^\gamma + F_{LT}^\gamma + F_{LL}^\gamma \simeq \frac{1}{x}F_2^\gamma + \frac{3}{2}F_L^\gamma$, where the indices T and L refer to the transverse and longitudinal photon, respectively. So far mostly studied are the unpolarized photon structure functions, F_2^γ and F_L^γ .

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Here I refer to the recent review articles on virtual photon structure functions, M. Krawczyk's talk at PHOTON 2000 [2], and M. Nisius's summary talk at PHOTON 2001 [3]. More recent and detailed reviews are those by M. Klasen [4] and by I. Schienbein [5].

Now in the last few years, there has been much theoretical interest in the photon's spin structure functions g_1^γ and g_2^γ . Especially, g_1^γ has attracted much attention because of its relevance to the axial anomaly just like the case of nucleon spin structure function. Namely we can write down the QCD prediction of the 1st moment of g_1^γ . These spin dependent photon structure functions can be studied from the polarized ep collider or more directly from the polarized e^+e^- collision.

The virtual photon target provides a good place to study the structure of the operators appearing in the operator product expansion of the current product, since the virtual photon matrix elements of the operators are perturbatively calculable.

2. STRUCTURE FUNCTIONS

Now there are 8 independent structure functions for the virtual photon-photon scattering illustrated in Fig.2. This process is described by the structure tensor

$$W_{\mu\nu\rho\tau}(p, q) = \frac{1}{\pi} \text{Im} T_{\mu\nu\rho\tau}(p, q) \quad (1)$$

where $T_{\mu\nu\rho\tau}$ is the amplitude for $\gamma(q) + \gamma(p) \rightarrow \gamma(q) + \gamma(p)$ given by

$$T_{\mu\nu\rho\tau}(p, q) = i \int d^4x d^4y d^4z e^{iq \cdot x} e^{ip \cdot (y-z)} \times \langle 0 | T [J_\rho(y) J_\mu(x) J_\nu(0) J_\tau(z)] | 0 \rangle \quad (2)$$

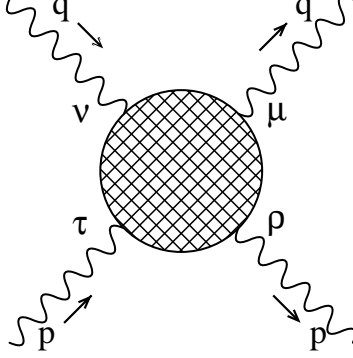


Figure 2. Virtual photon-photon scattering

where J is the electromagnetic current, and q and p are four-momenta of the probe and target photon, respectively.

For the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$ (Λ : QCD scale parameter) or $x^2 \ll (y-z)^2$ we have the Operator Product Expansion (OPE):

$$J_\mu(x)J_\nu(0) \sim \sum_n C_n(x)O_n(0) \quad (3)$$

where $O_n(0)$ and $C_n(x)$ are the spin- n operator and its coefficient function, respectively.

Now we can decompose the structure tensor following Budnev, Chernyak and Ginzburg [6] as

$$\begin{aligned} W^{\mu\nu\rho\tau}(p, q) = & (P_{TT})^{\mu\nu\rho\tau}W_{TT} + (P_{TT}^a)^{\mu\nu\rho\tau}W_{TT}^a \\ & + (P_{TT}^\tau)^{\mu\nu\rho\tau}W_{TT}^\tau + (P_{ST})^{\mu\nu\rho\tau}W_{ST} \\ & + (P_{TS})^{\mu\nu\rho\tau}W_{TS} + (P_{SS})^{\mu\nu\rho\tau}W_{SS} \\ & - (P_{TS}^\tau)^{\mu\nu\rho\tau}W_{TS}^\tau - (P_{TS}^a)^{\mu\nu\rho\tau}W_{TS}^a \end{aligned} \quad (4)$$

where P_i 's are the projectors given by [1,6]

$$(P_{TT})^{\mu\nu\rho\tau} = R^{\mu\nu}R^{\rho\tau} \quad \text{etc.} \quad (5)$$

with

$$\begin{aligned} R^{\mu\nu} = & -g^{\mu\nu} + \frac{1}{X} [w(q^\mu p^\nu + p^\mu q^\nu) \\ & - q^2 p^\mu p^\nu - p^2 q^\mu q^\nu] \\ w = & p \cdot q, \quad X = (p \cdot q)^2 - p^2 q^2 \end{aligned} \quad (6)$$

and satisfy

$$\begin{aligned} (P_{TT}) \cdot (P_{TT}) &= 4 \quad \text{etc.} \\ (P_i)^{\mu\nu\rho\tau}(P_j)_{\mu\nu\rho\tau} &= 0 \quad (i \neq j) \end{aligned} \quad (7)$$

where the subscripts T and S refer to the transverse and longitudinal photon, respectively.

We now introduce the s-channel helicity amplitudes as follows:

$$W(ab|a'b') = \epsilon_\mu^*(a)\epsilon_\rho^*(b)W^{\mu\nu\rho\tau}\epsilon_\nu(a')\epsilon_\tau(b') \quad (8)$$

where $\epsilon_\mu(a)$ represents the photon polarization vector with helicity a , and $a = 0, \pm 1$. From angular momentum conservation, parity conservation, and time reversal invariance [7], we have in total 8 independent helicity amplitudes, which we may take as

$$\begin{aligned} & W(1, 1|1, 1), \quad W(1, -1|1, -1), \quad W(1, 0|1, 0), \\ & W(0, 1|0, 1), \quad W(0, 0|0, 0), \\ & W(1, 1|-1, -1), \quad W(1, 1|0, 0), \quad W(1, 0|0, -1). \end{aligned} \quad (9)$$

The first 5 amplitudes are helicity-nonflip, while the last 3 amplitudes are helicity-flip.

Here we note the relation between the 8 structure functions (4) and the 8 helicity amplitudes (9), which are given by

$$\begin{aligned} W_{TT} &= \frac{1}{2} [W(1, 1|1, 1) + W(1, -1|1, -1)] \\ W_{ST} &= W(0, 1|0, 1) \\ W_{TS} &= W(1, 0|1, 0), \quad W_{SS} = W(0, 0|0, 0) \\ W_{TT}^a &= \frac{1}{2} [W(1, 1|1, 1) - W(1, -1|1, -1)] \\ W_{TT}^\tau &= W(1, 1|-1, -1) \\ W_{TS}^\tau &= \frac{1}{2} [W(1, 1|0, 0) - W(1, 0|0, -1)] \\ W_{TS}^a &= \frac{1}{2} [W(1, 1|0, 0) + W(1, 0|0, -1)]. \end{aligned} \quad (10)$$

Since the helicity-nonflip amplitudes are non-negative, the first four structure functions are positive-definite, while the last four are not. We also note that all structure functions are symmetric under interchange of $p \leftrightarrow q$ except W_{ST} and W_{TS} , which satisfy

$$W_{ST}(w, q^2, p^2) = W_{TS}(w, p^2, q^2). \quad (11)$$

3. $F_2^\gamma(x, Q^2, P^2)$ and $g_1^\gamma(x, Q^2, P^2)$

We now turn to the NLO QCD calculation of the virtual photon structure functions $F_2^\gamma(x, Q^2, P^2)$ and $g_1^\gamma(x, Q^2, P^2)$. The basic theoretical framework for the NLO QCD calculation is either based on the OPE supplemented by Renormalization Group (RG) or on the DGLAP type parton evolution equation. The master formula for the n -th moment for F_2^γ and g_1^γ has the same form, and for the g_1^γ case, we have for the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$:

$$\begin{aligned} \int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) &= \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \times \\ &\left[\sum_{i=+,-,NS} L_i^n \frac{4\pi}{\alpha_s(Q^2)} \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0+1} \right\} \right. \\ &+ \sum_{i=+,-,NS} \mathcal{A}_i^n \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0} \right\} \\ &+ \sum_{i=+,-,NS} \mathcal{B}_i^n \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0+1} \right\} \\ &\left. + \mathcal{C}^n + \mathcal{O}(\alpha_s) \right] \quad (12) \end{aligned}$$

where L_i^n , \mathcal{A}_i^n , \mathcal{B}_i^n and \mathcal{C}^n are the renormalization scheme independent coefficients computed from the one- and two-loop anomalous dimensions together with one-loop coefficient functions.

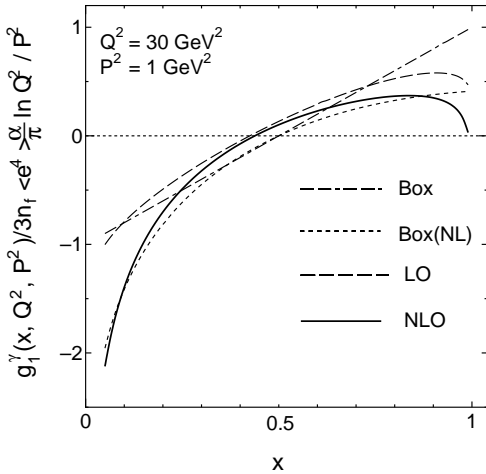


Figure 3. Spin structure function $g_1^\gamma(x, Q^2, P^2)$ for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$

$\alpha_s(Q^2)$ is the QCD running coupling constant, and λ_i^n ($i = +, -, NS$) denote the eigenvalues of one-loop anomalous dimension matrix. We have shown $g_1^\gamma(x, Q^2, P^2)$ in Fig.3, where the NLO g_1^γ corresponds to the solid line, for $n_f = 3$, $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$ with $\Lambda = 0.2 \text{ GeV}$.

For a real photon ($P^2 = 0$), the 1st moment vanishes to all orders of $\alpha_s(Q^2)$ in QCD [8]:

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0. \quad (13)$$

Now the question is what about the $n = 1$ moment of the virtual photon case. Taking $n \rightarrow 1$ limit of (12) the first 3 terms vanish as

$$L_i^n \rightarrow 0, \quad \sum_i \mathcal{A}_i^n \{ \} \rightarrow 0, \quad \sum_i \mathcal{B}_i^n \{ \} \rightarrow 0. \quad (14)$$

Denoting $\langle e^4 \rangle = \sum_{i=1}^{n_f} e_i^4 / n_f$ (e_i : the i -th quark charge, n_f : the number of active flavors), we have $\mathcal{C}^{n=1} = 12\beta_0 \langle e^4 \rangle (B_G^n + A_{qG}^n)|_{n=1}$. (15)

Here we note that the sum of the one-loop coefficient function B_G^n and the finite photon matrix element of quark operator A_{qG}^n is renormalization-scheme independent and equal to $-2n_f$ for $n = 1$.

Therefore we have

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} e_i^4 + \mathcal{O}(\alpha_s). \quad (16)$$

We can go a step further to $\mathcal{O}(\alpha_s)$ QCD corrections which turn out to be [9]

$$\begin{aligned} \int_0^1 dx g_1^\gamma(x, Q^2, P^2) &= -\frac{3\alpha}{\pi} \left[\sum_{i=1}^{n_f} e_i^4 \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right) \right. \\ &\quad \left. - \frac{2}{\beta_0} \left(\sum_{i=1}^{n_f} e_i^2 \right)^2 \left(\frac{\alpha_s(P^2)}{\pi} - \frac{\alpha_s(Q^2)}{\pi} \right) \right] \\ &\quad + \mathcal{O}(\alpha_s^2). \quad (17) \end{aligned}$$

This result coincides with the one obtained by Narison, Shore and Veneziano in ref.[10], apart from the overall sign for the definition of g_1^γ .

It would be extremely interesting to explain the transition from the vanishing sum rule (13) for the real photon to the non-vanishing result (16) or (17) for the virtual photon.

4. POSITIVITY CONSTRAINTS

Application of the Cauchy-Schwarz inequality [11,12] to the above photon helicity amplitudes leads to the following positivity bound [13]:

$$|W(a, b|a', b')| \leq \sqrt{W(a, b|a, b)W(a', b'|a', b')}. \quad (18)$$

More explicitly, we obtain the following three positivity constraints:

$$|W(1, 1| -1, -1)| \leq W(1, 1|1, 1) \quad (19)$$

$$|W(1, 1|0, 0)| \leq \sqrt{W(1, 1|1, 1)W(0, 0|0, 0)} \quad (20)$$

$$|W(1, 0|0, -1)| \leq \sqrt{W(1, 0|1, 0)W(0, 1|0, 1)}. \quad (21)$$

In terms of the structure functions (4), the positivity constraints read

$$|W_{TT}^\tau| \leq (W_{TT} + W_{TT}^a) \quad (22)$$

$$|W_{TS}^\tau + W_{TS}^{\tau a}| \leq \sqrt{(W_{TT} + W_{TT}^a)W_{SS}} \quad (23)$$

$$|W_{TS}^\tau - W_{TS}^{\tau a}| \leq \sqrt{W_{TS}W_{ST}}. \quad (24)$$

For the real photon target, $P^2 = 0$, the number of independent structure functions or helicity amplitudes reduces to four, which are [13]

$$\begin{aligned} W_{TT} &= \frac{1}{2}F_1^\gamma, \quad W_{ST} = \frac{1}{2x}F_L^\gamma, \quad W_{TT}^\tau = 2W_3^\gamma, \\ W_{TT}^a &= \frac{1}{2}g_1^\gamma \end{aligned} \quad (25)$$

and we have only one positivity constraint (22).

The virtual photon structure functions F_2^γ , F_L^γ , and g_1^γ are related to W_i 's as

$$\begin{aligned} F_1^\gamma(x, Q^2, P^2) &= 2 \left[W_{TT} - \frac{1}{2}W_{TS} \right] \\ F_2^\gamma(x, Q^2, P^2) &= \frac{2x}{\beta^2} \left[W_{TT} + W_{ST} - \frac{1}{2}W_{SS} - \frac{1}{2}W_{TS} \right] \\ F_L^\gamma(x, Q^2, P^2) &= \tilde{\beta}^2 F_2^\gamma - x F_1^\gamma, \\ g_1^\gamma(x, Q^2, P^2) &= \frac{2}{\beta^2} \left[W_{TT}^a - \frac{(P^2 Q^2)^{1/2}}{w} W_{TS}^{\tau a} \right] \\ g_2^\gamma(x, Q^2, P^2) &= -\frac{2}{\beta^2} \left[W_{TT}^a - \frac{w}{(P^2 Q^2)^{1/2}} W_{TS}^{\tau a} \right] \\ \text{with } \tilde{\beta} &= \left(1 - \frac{P^2 Q^2}{w^2} \right)^{1/2} \text{ and } w = p \cdot q. \end{aligned} \quad (26)$$

For $\tilde{\beta} \approx 1$

$$\begin{aligned} W_{TT}(x, Q^2, P^2) &\approx \frac{1}{2}F_1^\gamma(x, Q^2, P^2) \\ &= \frac{1}{2x} \{ F_2^\gamma(x, Q^2, P^2) - F_L^\gamma(x, Q^2, P^2) \} \\ W_{TT}^a(x, Q^2, P^2) &\approx \frac{1}{2}g_1^\gamma(x, Q^2, P^2). \end{aligned} \quad (27)$$

Now let us see if the first positivity constraint (22) is satisfied in the pQCD results.

$$\begin{aligned} |W_{TT}^\tau(x, Q^2, P^2)| &\leq \frac{1}{2} [F_1^\gamma(x, Q^2, P^2) + g_1^\gamma(x, Q^2, P^2)]. \end{aligned} \quad (28)$$

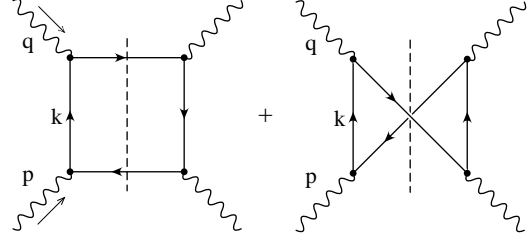


Figure 4. The Box diagram contributions

The parton model Box diagram (Fig.4) calculation for $W_{TT}^\tau(x, Q^2, P^2) = 2W_3^\gamma$ gives the LO QCD result up to $\mathcal{O}(\alpha_s)$ as

$$\begin{aligned} W_{TT}^\tau(x, Q^2, P^2) &= \frac{\alpha}{2\pi} \delta_\gamma [(-2x^2) + \mathcal{O}(\alpha_s(Q^2))]. \end{aligned} \quad (29)$$

In Fig. 5 we have plotted the NLO QCD result of $\frac{1}{2}[F_1^\gamma + g_1^\gamma]$ and the LO result of $|W_{TT}^\tau|$ as functions of x for the case $P^2/Q^2 = 1/30$ with the number of active flavors, $n_f = 3$.

We observe that the inequality (28) is satisfied for almost all the allowed x region except near $x_{\max} = 1/(1 + P^2/Q^2) (\approx 0.968 \text{ for } P^2/Q^2 = 1/30)$. The reason of why the inequality is not satisfied near x_{\max} is as follows. We notice that the graph of $\frac{1}{2}(F_1^\gamma + g_1^\gamma)$ falls rapidly as $x \rightarrow x_{\max}$. In the QCD parton picture, this is due to the total momentum conservation of the partons inside the photon. From the large n behavior of the structure functions F_1^γ and g_1^γ , they vanish like $-1/\ln(1-x)$ as $x \rightarrow 1$. The NLO QCD effects further suppress F_1^γ and g_1^γ at large x . While, the

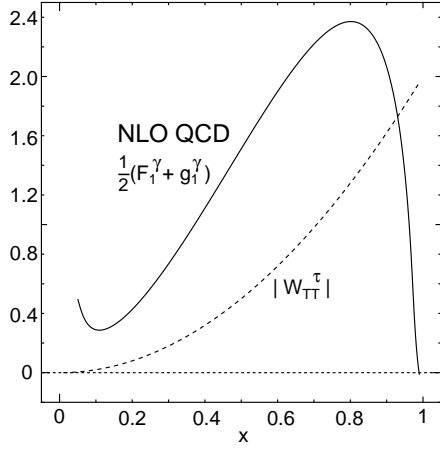


Figure 5. NLO QCD result of $\frac{1}{2}[F_1^\gamma + g_1^\gamma]$ vs. LO result of W_{TT}^τ in units of $(\alpha/2\pi)\delta_\gamma$, for $Q^2 = 30\text{GeV}^2$, $P^2 = 1\text{GeV}^2$ with $\Lambda = 0.2\text{GeV}$ and $n_f = 3$.

LO QCD result of W_{TT}^τ is just same as the massless quark parton model, where the power corrections of P^2/Q^2 is neglected. Hence $|W_{TT}^\tau|$ in Fig. 5 increases as a function of x like x^2 and violates the inequality in the vicinity of x_{max} . The *physical* W_{TT}^τ should vanish as $x \rightarrow x_{\text{max}}$, which is realized in the case of parton model with massive quark. This implies the necessity of introducing the quark mass effects for the photon coefficient function.

5. CONCLUSION

To summarize we have investigated the aspects of the structure functions of the virtual photon for the most general case. The virtual photon structure could be studied in future ep and e^+e^- colliders and provides a good testing ground for the structure of the current product.

We have discussed the sum rule for the g_1^γ for the virtual photon target. It would be an intriguing question how we can understand the transition from vanishing 1st moment for real photon to non-vanishing one for virtual photon.

For the eight independent structure functions

of the virtual photon we have derived three positivity constraints and discussed if the first inequality is satisfied in the QCD parton model computation of F_1^γ , g_1^γ and W_{TT}^τ .

For another polarized structure function g_2^γ , the twist-3 effects are important, where we have to solve the mixing problem for the full QCD analysis [14]. There remain a number of virtual photon structure functions yet to be studied.

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